ToCA GR Bridge v1.0

How General Relativity Emerges from the Substrate

ToCA Companion Document — Connects v5.0 to Einstein's Field Equations

Henrik Lehn — Independent Researcher, Copenhagen

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1 Purpose

General Relativity (GR) is the most precisely tested theory of gravity. Any theory of fundamental physics must either reproduce GR or explain why it appears to hold. ToCA does the former: GR emerges as the macroscopic description of tension gradients in the substrate.

This document establishes the mapping between ToCA's substrate variables and GR's geometric quantities. It does not "derive" GR from scratch — it shows that GR's equations are the natural language for describing what the substrate does at macroscopic scales.

What this document does:

- Maps each GR quantity to a substrate variable
- Shows why Einstein's field equations describe tension dynamics
- Explains time dilation, geodesics, and the cosmological constant in ToCA language
- Identifies what is derived, what is assumed, and what remains open

What it does not do:

- Quantize gravity (that requires the full substrate dynamics, not the macroscopic limit)
- Replace GR (GR is correct at macroscopic scales; ToCA explains *why* it is correct)

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2 The Core Idea

In GR, spacetime is a smooth manifold with a metric $g_{\mu\nu}$ that encodes distances and curvature. Matter tells spacetime how to curve; spacetime tells matter how to move.

In ToCA, there is no spacetime. There is a substrate — Grundstof 0 — with tension that can be locked or relaxed. The pattern of locked tension creates gradients. Those gradients are what we experience as gravity and curved spacetime.

The metric $g_{\mu\nu}$ is not fundamental. It is the macroscopic average of the substrate's tension field, smoothed over many nodes.

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3 The Mapping

3.1 Metric tensor ↔ Tension configuration

The metric tensor $g_{\mu\nu}$ describes distances between events in spacetime. In ToCA, "distance" between two substrate regions is determined by the tension configuration between them.

> $$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

where $\eta_{\mu\nu}$ is the flat Minkowski metric (the substrate in its relaxed ground state, Grundstof 0 with $\nabla\Phi = 0$) and $h_{\mu\nu}$ is the perturbation caused by locked tension.

In ToCA: $h_{\mu\nu}$ is proportional to the local tension deviation from the floor:

> $$h_{\mu\nu}(x) \propto D(x) - D_{floor}$$

More locked tension → larger $h_{\mu\nu}$ → more deviation from flat space. No locked tension → $h_{\mu\nu} = 0$ → flat spacetime. This is the weak-field limit, valid where $D - D_{floor}$ is small compared to D_{floor}.

3.2 Ricci curvature ↔ Tension gradient

The Ricci curvature tensor $R_{\mu\nu}$ measures how spacetime curvature focuses or defocuses nearby geodesics. In ToCA, this curvature is the gradient of tension — how rapidly the tension field changes from one region to another.

> $$R_{\mu\nu} \propto \nabla_\mu \nabla_\nu D$$

Curvature exists wherever the tension field has a non-zero second derivative. Near a massive object (high locked tension), D falls off with distance, creating positive curvature. In empty space far from mass, D is nearly constant, and curvature is nearly zero.

3.3 Stress-energy tensor ↔ Locked tension density

The stress-energy tensor $T_{\mu\nu}$ describes the distribution of energy and momentum. In ToCA, energy IS locked tension. The stress-energy tensor maps directly to the locked tension density:

> $$T_{\mu\nu} \propto \rho_{locked}(x) \times u_\mu u_\nu + \text{pressure terms}$$

where ρ_{locked} is the density of locked tension (frozen nodes per volume) and u_μ is the bulk flow of locked configurations. The pressure terms arise from the interaction between locked and relaxed sectors — mediated by $\beta = f(1-f)$.

Mass-energy equivalence: $E = mc^2$ is the statement that locked tension (mass) and total tension (energy) are the same quantity in different configurations. A locked configuration with mass m contains tension mc^2, which is the amount of substrate tension locked into that configuration.

3.4 Cosmological constant ↔ Residual tension floor

The cosmological constant Λ represents a uniform energy density pervading all of space. In ΛCDM, it is a free parameter. In ToCA, it is D_floor — the residual tension that can never be eliminated (Axiom 5).

> $$\Lambda = (8\pi G/c^4) \times D_{floor}$$

D_floor exists because the substrate always has boundary frustration between frozen and relaxed regions (Section 3 of Core Dynamics). It is not a free parameter — it emerges from the FCC topology. And it is not constant across cosmic history — it depends on f_frozen(n), which evolves.

This explains "dark energy": The accelerating expansion is not driven by a mysterious energy. It is driven by D_floor — the irreducible tension in the substrate. As f_frozen increases (more structure forms), D_floor adjusts, and the effective Λ changes. DESI's evidence for evolving dark energy ($w_0 > -1$, $w_a < 0$) is exactly what ToCA predicts: Λ is not constant because D_floor is not constant.

3.5 Geodesics ↔ Tension-minimising paths

In GR, free particles follow geodesics — paths that extremise proper time. In ToCA, a locked configuration moves through the substrate along the path that minimises its total tension interaction. This is not a force. It is the natural consequence of tension gradients.

> **A "falling" object is not pulled by gravity. It is following the path of least tension resistance through the substrate.**

The geodesic equation:

> $$d^2x^\mu/d\tau^2 + \Gamma^\mu_{\alpha\beta} (dx^\alpha/d\tau)(dx^\beta/d\tau) = 0$$

is the statement that locked configurations follow the substrate's tension field without generating additional gradients. The Christoffel symbols $\Gamma^\mu_{\alpha\beta}$ encode the local tension gradient, and the equation says: move so as not to add to it.

3.6 Time dilation ↔ Latency

In GR, clocks run slower in gravitational fields. In ToCA, this is latency (Law 4): processing speed depends on local locked tension density.

> $$d\tau/dt = \sqrt{(1 - 2GM/rc^2)} \approx \sqrt{(1 - h_{00})} \approx \sqrt{(1 - (D - D_{floor})/D_{max})}$$

Near a massive object, D is high (much locked tension). Processing is slow (high latency). Clocks run slow. Far from mass, D is near D_floor, processing is fast, clocks run normally.

This is not an analogy. It is the mechanism. Time dilation IS substrate latency. And it follows directly from Axiom 1 (discrete processing) and the tension field.

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4 Einstein's Field Equations in ToCA Language

4.1 The equations

Einstein's field equations:

> $$G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)\, T_{\mu\nu}$$

In ToCA language, each term has a substrate interpretation:

- **$G_{\mu\nu}$** (Einstein tensor) = the net curvature of the tension field, encoding how gradients focus or defocus
- **$\Lambda g_{\mu\nu}$** = the residual floor contribution (D_floor × metric)

- **$T_{\mu\nu}$** = locked tension density and its flow

The equation says: **the curvature of the tension field (left side) is determined by the distribution of locked tension (right side), plus the irreducible floor.**

This is not surprising — it is what Axiom 4 (tension minimisation) requires. The substrate arranges its tension field to minimise global D, subject to the constraint that locked configurations exist. The field equations describe the optimal arrangement.

4.2 Why $G_{\mu\nu}$ and not something else?

The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the unique tensor that satisfies $\nabla_\mu G^{\mu\nu} = 0$ (the Bianchi identity). This conservation law corresponds to Axiom 3 in ToCA: total tension is conserved (LT + RT = 1). The Bianchi identity IS tension conservation expressed in geometric language.

The fact that Einstein's equations have this specific form — and not some other combination of curvature tensors — follows from conservation. In ToCA, conservation is an axiom. In GR, it is a geometric identity. They are the same statement in different languages.

4.3 Newton's limit

In the weak-field, slow-motion limit:

> $$\nabla^2\Phi = 4\pi G\rho$$

where Φ is the gravitational potential and ρ is mass density. In ToCA:

> $$\nabla^2 D = (\text{constant}) \times \rho_{locked}$$

The Laplacian of the tension field equals the locked tension density. This is Poisson's equation — the statement that tension gradients are sourced by locked tension. It follows from diffusion (Axiom 4 operating locally) reaching equilibrium around a concentration of locked tension.

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5 Gravitational Waves ↔ Tension Waves

Gravitational waves are propagating perturbations of the metric: $h_{\mu\nu}$ travelling at speed c. In ToCA, these are tension waves — disturbances in the substrate's tension field propagating through the FCC lattice.

The speed of propagation is c — the substrate's maximum update speed. This is not a coincidence; c is set by the lattice spacing (l_p) and the update rate ($1/t_p$):

> **$c = l_p / t_p$**

Gravitational waves carry energy because they represent tension being transported through the substrate. Their quadrupolar nature follows from the fact that tension is a scalar (not a vector), so the leading-order radiation is quadrupole, not dipole.

> **LIGO's detection of gravitational waves is a detection of tension waves in the substrate.**

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6 Black Holes ↔ Maximal Locking

A black hole is a region where the tension field is so extreme that nothing can escape — not even light (tension waves). In ToCA, this is a region of maximal locked tension, where:

- $D \to D_{max}$ (tension at its maximum)
- Latency → maximum (processing nearly halts)
- The effective metric develops a horizon: beyond it, all tension-minimising paths lead inward

The singularity problem in GR (where curvature becomes infinite) does not arise in ToCA, because the substrate is discrete (Axiom 1). There is a minimum length (l_p) and a maximum tension (D_{max}). The "singularity" is a region of maximum locked tension at the Planck scale — extreme but finite.

> **OPEN:** The precise internal structure of a black hole in ToCA (distribution of locked tension, information storage, Hawking radiation mechanism) has not been worked out. This is a major open problem.

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7 Expansion ↔ Tension Processing

Cosmic expansion in GR is described by the Friedmann equations, which relate the expansion rate $H(t)$ to the energy content. In ToCA, expansion is relaxed tension processing itself outward — the substrate reducing its tension by spreading it over a larger volume.

The Friedmann equation:

> **$H^2 = (8\pi G/3)(\rho_m + \rho_\Lambda) - k/a^2$**

maps to:

> **$H^2 \propto f_{frozen} \times (\text{tension per locked node}) + D_{floor}$**

The expansion rate depends on how much locked tension exists (f_frozen), how concentrated it is (tension per node), and the residual floor (D_floor). This is the content of our α-integral analysis in Core Dynamics.

The accelerating expansion (Λ-dominated era) occurs when D_floor dominates over the matter term — when most tension has been processed and only the irreducible floor remains. This is where our universe is now: $f_{frozen} \approx 0.31$, and the floor dominates ($\alpha \approx 0.69$).

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8 What ToCA Adds to GR

GR describes gravity perfectly at macroscopic scales. ToCA does not change GR's predictions — it explains *why* they are what they are:

1. Why does mass curve spacetime? Because locked tension creates gradients in the substrate's tension field. Curvature is the gradient structure.

2. Why do objects fall? Because locked configurations follow tension-minimising paths. Falling is not a force — it is tension equalisation.

3. Why does time dilate in gravity? Because substrate processing slows in regions of high locked tension. Latency IS time dilation.

4. Why is c the speed limit? Because $c = l_p/t_p$ is the substrate's maximum update rate. Nothing can propagate faster than one node per Planck time.

5. Why does Λ exist? Because $D_{floor} > 0$ always (Axiom 5). The vacuum is not empty — it is the substrate at its minimum tension. And it is not constant — it evolves with f_frozen(n).

6. Why do gravitational waves travel at c? Because they are tension waves in the substrate, and c is the maximum propagation speed of the lattice.

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9 What GR Cannot Do That ToCA Can (In

Principle)

Quantum gravity. GR fails at the Planck scale because it assumes a smooth manifold. ToCA's substrate is discrete (Axiom 1), so there is no singularity problem. A future theory of quantum gravity in ToCA would describe tension fluctuations at the node level.

Unification. GR describes gravity but not the other forces. In ToCA, all forces are aspects of tension: gravity = tension gradients, electromagnetism = tension twist, strong force = deep locking configurations. This unification is conceptual, not yet quantitative.

Dark matter. GR treats dark matter as an unknown particle. ToCA identifies it as gradient-locked tension (11/13 of all locked tension) — the same substance as baryonic matter but locked through a different mechanism.

Dark energy evolution. GR's Λ is constant. ToCA's D_floor evolves with f_frozen(n). DESI's evidence for $w_0 > -1$ supports this.

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10 Open Problems

> **OPEN:** The quantitative mapping h_μν = f(D) beyond weak field. What is the exact functional relationship between the substrate's tension field and the metric perturbation? The weak-field limit (h ∝ D – D_floor) is clear; the strong-field regime (near black holes) requires a non-linear extension.

> **OPEN:** The value of G. Newton's gravitational constant relates tension gradients to locked tension density. In ToCA, G should be derivable from the FCC lattice spacing, the Planck units, and the coupling structure. This has not been done.

> **OPEN:** Frame dragging and spin. Rotating locked configurations drag the substrate, creating off-diagonal metric components. The mechanism (locked twist in rotation) is identified but not quantified.

> **OPEN:** Gravitational wave polarisation. GR predicts two tensor polarisations (+ and ×). ToCA should predict the same from the FCC lattice's symmetries, but this has not been verified.

> **OPEN:** Black hole interior. The distribution of locked tension inside a horizon, the mechanism of Hawking radiation, and the resolution of the information paradox in ToCA are all unresolved.

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11 Yin and Yang: Why ToCA Needs GR and GR Needs ToCA

ToCA and GR are not competitors. They are complements — two descriptions of the same reality that cannot exist independently.

GR without ToCA is a geometry without a cause. It describes *how* spacetime curves but not *why*. It says mass-energy curves spacetime — but what is mass? What is energy? Why does curvature exist? GR has no answer. It accepts the metric as fundamental and works from there. The cosmological constant Λ is inserted by hand. Dark matter is postulated. Dark energy is a label for ignorance.

ToCA without GR is a substrate without a language for large-scale geometry. It describes *why* tension locks, *why* gradients form, *why* processing slows near mass — but it cannot compute orbits, predict light bending, or describe the expansion history without borrowing GR's mathematical framework. The field equations are the only proven language for translating substrate dynamics into observable predictions at macroscopic scales.

Together they form a complete picture:

- ToCA provides the *substance*: what the universe is made of (Grundstof 0), why structure forms (tension minimisation), what dark matter is (gradient-locked tension), why Λ exists (D_floor), and why time dilates (latency).
- GR provides the *geometry*: how to compute distances, how to predict orbits, how to describe expansion, how to extract observables from the tension field.

Neither is more fundamental than the other. The substrate creates the geometry. The geometry describes the substrate. Remove either one and you have half a theory.

This is why the document is called a "Bridge" — not a derivation, not a replacement, but a connection between two necessary perspectives on the same reality.

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12 Conclusion

General Relativity is not replaced by ToCA. It is explained by ToCA.

GR's equations describe the macroscopic behaviour of the substrate's tension field — how gradients form around locked tension, how processing speed varies with tension density, and how the residual floor drives expansion.

Every quantity in GR has a substrate counterpart:

GR quantity	**ToCA substrate**
Metric $g_{\mu\nu}$	Tension configuration
Curvature $R_{\mu\nu}$	Tension gradient ($\nabla^2 D$)
Stress-energy $T_{\mu\nu}$	Locked tension density
Cosmological constant Λ	Residual floor D_{floor}
Geodesic	Tension-minimising path
Time dilation	Processing latency
Speed of light c	Maximum update rate l_p/t_p
Gravitational waves	Tension waves
Black hole	Maximal locking region

The mapping is consistent, physical, and introduces no new parameters. It derives its authority from the six axioms in v5.0.

Gravity is not a force. It is what tension gradients look like from the inside.