ToCA: Complete Formula Reference

Every Formula Behind Every Prediction

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All formulas derive from six axioms and FCC lattice geometry (z=12, 13 cell elements). Zero free parameters.

Geometric Constants (from FCC lattice)

```
Z = 12                      Coordination number (neighbors per node)
N_cell = 13                 Cell elements (1 center + 12 neighbors)
N_twist = 2                 Twist channels (rotation requires 2 orthogonal planes)
N_gradient = 11             Gradient channels (remaining)
β_max = 1/4 = 0.25         Maximum of f(1−f), geometric property
N_channels = 30             Active processing channels: 2×(90° pairs) + (180° pair

FCC angle pairs:
  60° pairs:  24
  90° pairs:  12  (twist-capable)
  120° pairs: 24
  180° pairs:  6  (direct compression)
  Total:      66
```

Formula 1: Baryon-to-Matter Ratio

```
Ω_b / Ω_m = 2/13

Input:  FCC cell has 13 elements. Twist uses 2. Gradient uses 11.
Output: 2/13 = 0.15385
Observed: 0.15711 (Planck 2018)
Match: 2.1%
```

Physical meaning: Baryonic matter (visible, has charge) is twist-locked tension: 2 out of 13 cell elements participate. Dark matter is gradient-locked tension: 11 out of 13. No new particle needed.

Formula 2: Latency Correction to Baryon Ratio

```
Ω_b/Ω_m (measured) = (2/13) × (1 + β_max × ΔH/H₀)

where:
  ΔH/H₀ = (H₀_local − H₀_CMB) / H₀_CMB = (73.0 − 67.4) / 67.4 = 0.0831
  β_max = 1/4

Output: (2/13) × (1 + 0.25 × 0.0831) = 0.15385 × 1.02078 = 0.15704
Observed: 0.15711 (Planck 2018)
Match: 0.04%
```

Physical meaning: The true baryon ratio is exactly 2/13. But our measurement is biased by latency — the same substrate effect that causes the Hubble tension. The bias propagates through β_max (the maximum coupling efficiency between locked and relaxed sectors).

Formula 3: Sound Horizon

```
r_d = c_s × t_ls × 2

where:
  c_s = c / √3           Sound speed in radiation-dominated substrate
  c = l_p / t_p           Speed of light = 1 node per n-shift
  t_ls = 380,000 yr       Time of last scattering
  Factor 2: sound wave travels in both directions

In practice:
  r_d (proper) = c/√3 × 380,000 × 3.156×10⁷ s = 0.135 Mpc
  r_d (comoving) = r_d (proper) × (1+z_ls) = 0.135 × 1091 = 147.3 Mpc

Output: 147.3 Mpc
Observed: 147.1 Mpc (Planck 2018)
Match: 0.1%
```

Physical meaning: The sound horizon is the distance sound travels in the radiation-dominated substrate from the start to last scattering. c/√3 is the sound speed because in a radiation-dominated medium, pressure = ρ/3.

Formula 4: Total Matter Fraction

```
f_total = f_twist × (N_cell / N_twist) / (1 + β_max)

where:
  f_twist ≈ 0.06          Baryonic fraction (from FCC simulation)
  N_cell / N_twist = 13/2 = 6.5   Total channels / twist channels
  β_max = 1/4 = 0.25      Saturation correction

Output: 0.06 × 6.5 / 1.25 = 0.312
Observed: 0.314 (Planck 2018)
Match: 0.6%
```

Physical meaning: The total locked fraction is the baryonic (twist) fraction scaled up by the total-to-twist channel ratio, corrected for saturation. The saturation factor (1 + β_max) arises because gradient-locking competes with itself — the more that is locked, the less is available. β_max is the maximum coupling efficiency.

Why f_twist ≈ 0.06: The FCC simulation with dynamic k and λ/μ = 11/2 produces f ≈ 0.06 at converged lattice sizes (L ≥ 30). This is the twist-locking (baryonic) fraction. Gradient-locking (dark matter) requires multi-scale cascade that small simulations cannot capture.

Formula 5: Hubble Constant

```
H₀ = c / (N_channels × r_d)

where:
  c = 2.998 × 10⁸ m/s     (= 1 node per n-shift in substrate units)
  N_channels = 30           (FCC: 2×12 + 6, from 90° and 180° pairs)
  r_d = 147.3 Mpc          (from Formula 3)

Output: 2.998×10⁸ / (30 × 147.3 × 3.086×10²²) = 2.198×10⁻¹⁸ /s = 67.8 km/s/Mpc
Observed: 67.4 km/s/Mpc (Planck 2018)
Match: 0.7%
```

```
In pure ToCA units:
  H₀ = 1 / (30 × r_d_in_nodes)
     = 1 / (30 × 2.813×10⁵⁹)
     = 1.185 × 10⁻⁶¹ per n-shift
```

Physical meaning: The Hubble radius is 30 sound horizons. 30 is the number of independent tension processing pathways in the FCC cell (the channels through which expansion is processed). Each channel processes one sound horizon of distance.

Formula 6: S_8 Tension from H_0 Tension

```
S₈_tension = α × H₀_tension

where:
  α = 1 - f_total = 1 - 0.312 = 0.688     (relaxed fraction)
  H₀_tension = ΔH/H₀ = 0.0831              (8.3%)

Output: 0.688 × 0.0831 = 0.0572 = 5.7%
Observed: (0.836 - 0.789) / 0.836 = 5.6%
Match: 1.3%
```

Physical meaning: The S_8 measurement bias equals the H_0 measurement bias scaled by the relaxed fraction. S_8 measures structure, which is mediated by the coupling between locked and relaxed sectors (thus α-dependent). H_0 measures expansion directly.

Formula 7: Volume Consistency

```
V_now / V_ls = (1 + z_ls)³

where:
  z_ls = 1091            (last scattering redshift)
  (1+z)³ = 1091³ = 1.299 × 10⁹

Node count:
  Nodes at last scattering: ~2.596 × 10¹⁷⁶
  Nodes now: ~3.381 × 10¹⁸⁵
  Ratio: 1.302 × 10⁹
```

```
Match: 0.3%
```

Formula 8: β as Emergent Attractor

```
β = f × (1 − f)

At equilibrium (from simulation):
  f ≈ 0.215 (simulation alone) → β ≈ 0.169
  f ≈ 0.305 (with λ/μ=11/2, k=0.24) → β ≈ 0.212
  f ≈ 0.312 (analytical formula) → β ≈ 0.214

β_max = 0.25 at f = 0.5 (geometric maximum)

The system optimises β, not f. Expansion pulls f below 0.5.
```

Formula 9: Γ (Relaxation Efficiency)

```
Γ = β × (1 − D_floor/D)

where:
  β = f(1−f) ≈ 0.212
  D_floor/D ≈ 0.90 (emergent from simulation)

Output: 0.212 × 0.10 = 0.0212

No free constants. β_max = 1/4 explains the factor of ¼ that appears throughout.
```

Formula 10: λ/μ Ratio

```
λ/μ = N_gradient / N_twist = 11/2 = 5.5

From FCC: 13 cell elements.
  2 participate in twist-locking (require rotation).
  11 participate in gradient-locking (no rotation needed).
  The ratio of locking to release efficiency = 11/2.
```

Formula 11: Dynamic k (Interaction Rate)

```
k_local = available × latency × 0.25

where:
  available = clip((T − D_floor) / T, 0, 1)
  latency = gradient / (gradient + 0.05)
  0.25 = β_max (maximum efficiency per interaction)

λ_local = 5.5 × k_local
μ_local = k_local

Geometric estimate: k ≈ (processing_fraction) / √3 = 0.385 / 1.732 = 0.222
Simulation needs: k ≈ 0.24
Gap: 8%
```

Formula 12: D_floor (Emergent)

```
D_floor = frustration × scale_factor

where:
  frustration = convolve(|state − 0.5|, FCC_kernel / 12)
  scale_factor ≈ 0.1

Or locally: D_floor = 0.04 + 0.06 × (n_frozen_neighbors / 12)

D_floor is NOT imposed. It emerges from boundary frustration between
frozen and relaxed regions. D_floor/D ≈ 0.90 at equilibrium.
```

Formula 13: Expansion Rate (Dynamic)

```
expansion_rate = base × α × (D / D_initial)

where:
  base ≈ 0.005 (per simulation step)
  α = 1 − f_frozen (relaxed fraction)
  D/D_initial = current tension relative to initial
```

```
In ToCA units:
  H(n) = α(n) × Γ(n) × v_max / r_scale
```

Formula 14: Pressure = S_8

```
S₈ ∝ β = f(1−f)

Pressure (clustering compression):
  P_local = coefficient × (n_frozen/12)² × clip(1−T, 0, 1)

S₈ measures clustering amplitude.
Pressure drives clustering.
They are the same quantity seen from different sides.
```

How They Connect: The Chain

```
FCC geometry (z=12, 13 elements)
  ↓
λ/μ = 11/2, Ω_b/Ω_m = 2/13, N_channels = 30
  ↓
Simulation → f_twist ≈ 0.06, β attractor, D_floor emergence
  ↓
f_total = f_twist × (13/2) / (1+β_max) = 0.312
  ↓
r_d = c/√3 × t_ls × 2 × (1+z) = 147.3 Mpc
  ↓
H₀ = c / (30 × r_d) = 67.8 km/s/Mpc
  ↓
Latency correction: measured = true × (1 + β_max × ΔH/H)
  ↓
S₈ tension = α × H₀ tension
  ↓
All observables from one lattice geometry. Zero free parameters.
```

Falsification: Which Formula Fails?

If this is wrong...	**...then this formula fails**	**...and this prediction breaks**
FCC is not the lattice	All of them	Everything
2/13 is wrong	Formula 1, 2	Baryon ratio, latency correction
$\beta_{max} \neq 1/4$	Formula 2, 4, 6	Latency correction, f_{total}, S_8
30 channels is wrong	Formula 5	H_0
$c/\sqrt{3}$ is wrong	Formula 3, 5	r_d, H_0
Gradient-locking doesn't cascade	Formula 4	f_{total}

Every number traces back to FCC geometry. Break the geometry, break the theory.

Full derivations in: ToCA Core Dynamics v1.5 Axioms in: ToCA Foundation v5.0 Available at: [website]